Last time : Step ① $N \in \{0, 1, \infty\}$

Step ② : Rule out $N = \infty$

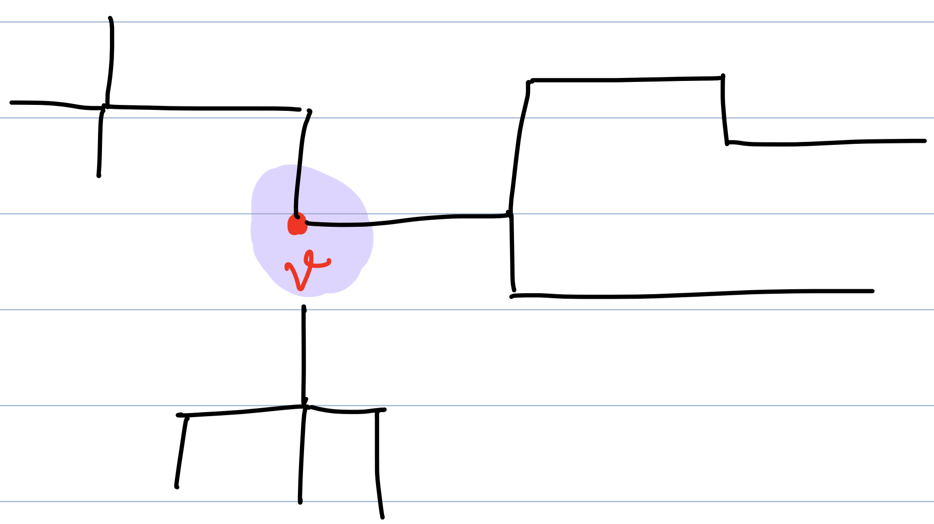
Burton and Keane

$$\mathbb{P}_p(N = \infty) \qquad , \; p \in (0,1)$$

Def^n: A vertex $v \in \mathbb{Z}^d$ is called an encounter point if

(i) $\# C(v) = \infty$

(ii) $C(v) \setminus \{v\}$ has no finite connected component and exactly 3 unbounded connected component.

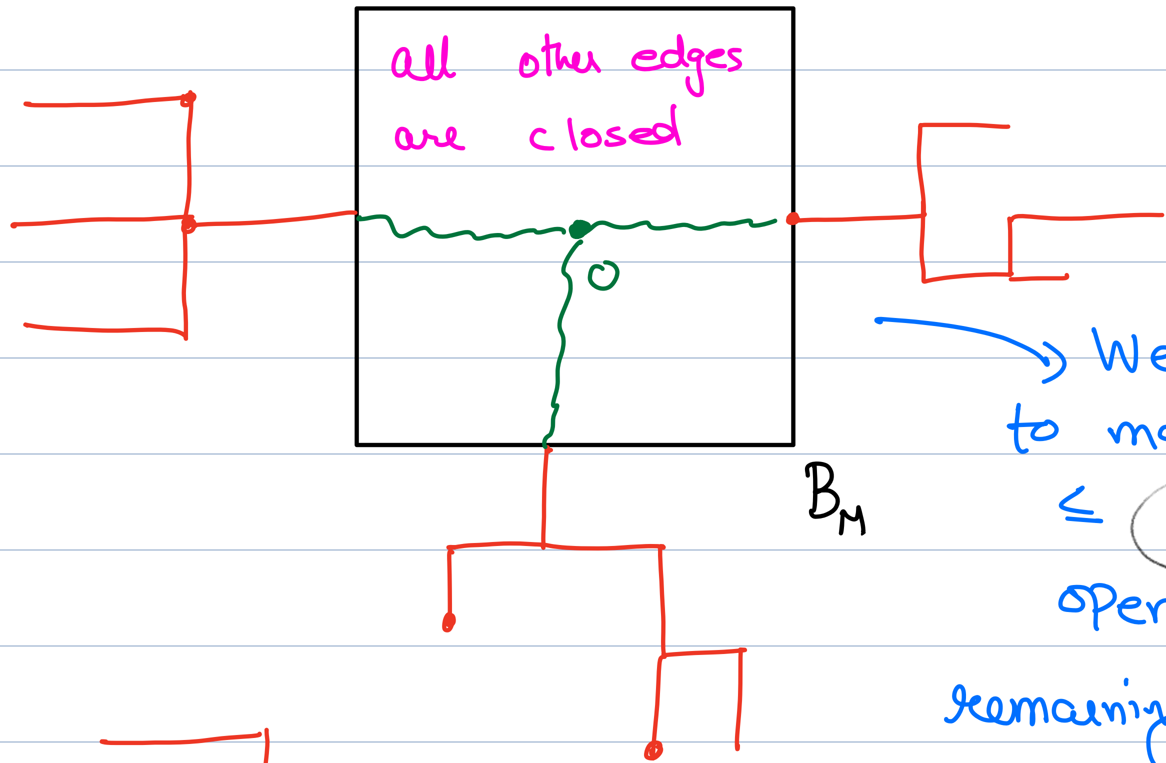


Suppose $\mathbb{P}_p(N = \infty) > 0$. Get M large enough s.t. $\eta > 0$

$\mathbb{P}_p\left(B_n \text{ intersects atleast 3 distinct unbounded open clusters}\right) > \eta$

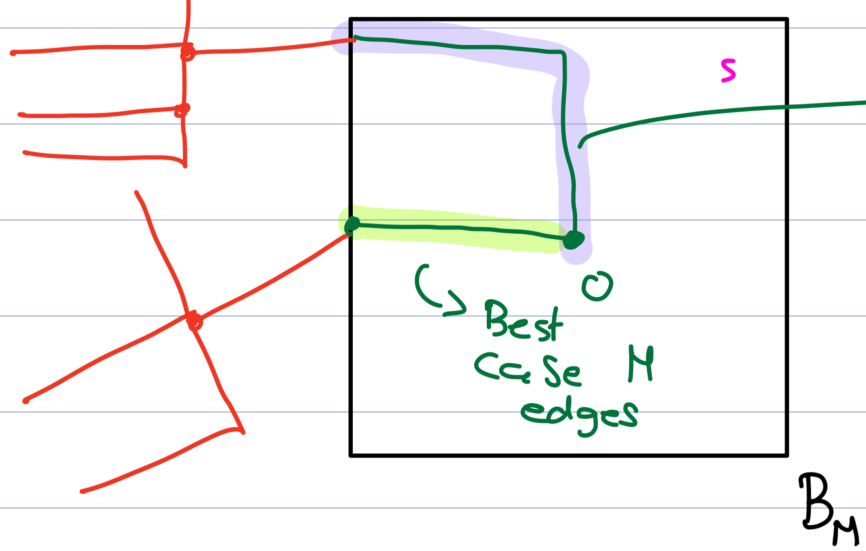
A_m

for all $n \geq M$



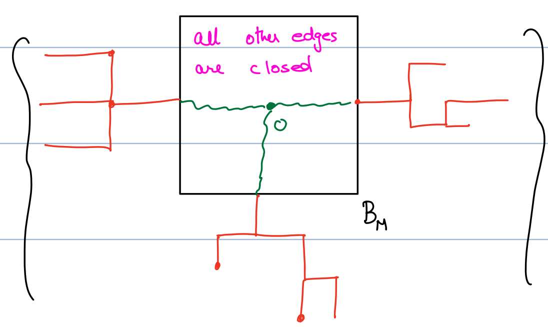
We need to make $\leq$ 6M edges open and remaining closed (in 2d)

3dM (in d-dim)



worst case 2M edges needed open.

Let



$=: F_M$

$$\mathbb{P}(F_M) \geq p^{3dM}(1-p)^{(2M)^d} =: \varepsilon > 0$$

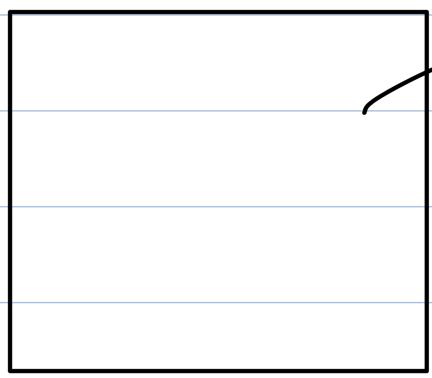
$$\mathbb{P}(A_M \cap F_M) \geq \mathbb{P}_p(A_M)\,\mathbb{P}_p(F_M)$$

$$\geq \varepsilon \cdot \eta =: \delta > 0$$

$$\Rightarrow \mathbb{P}_p(0 \text{ is an encounter point}) \geq \underbrace{\delta > 0}_{\text{indep of } M}.$$

for $p \in (0,1)$

By T.I., $\mathbb{P}_p(v \text{ is an encounter point}) \geq \delta > 0$

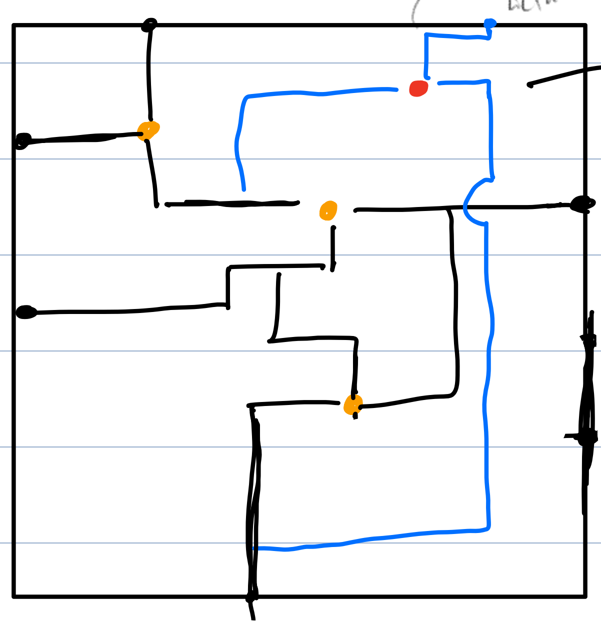


$(2\ell+1)^d$ many

B_ℓ

$\mathbb{E}_p(\#\text{ encounter pts in } B_\ell)$

$\geq \delta(2\ell+1)^d$



not actually an enc pt, just for figure

each new encounter pt has to give rise to a new boundary pt.

Lemma: If there are k-encounter points in a box B, then there are atleast $k+2$ vertices on the boundary δB of B which are connected by open paths to the encounter points.

$$\mathbb{E}_p\left(\begin{array}{l}\#\text{ points of vertices in } \delta B_\ell \text{ which} \\ \text{are connected to the encounter points}\end{array}\right)$$

$$\geq \delta(2\ell+1)^d + 2$$

$\left(\text{in particular } \geq \delta(2\ell+1)^d\right.$

B_ℓ

$(2\ell)^2$ — area

$\leq 8\ell$ — Perimeter

In d dimensions

$(2\ell+1)^d$, $\quad 2d(2\ell)^{d-1}$

Get ℓ large s.t. $\delta(2\ell+1)^d > 2d(2\ell)^{d-1}$

$\Rightarrow\Leftarrow$

Thus $\{N=\infty\}$ is not possible.

[Basically we embedded a tree in $\mathbb{Z}^d$ which cannot be done.]

Next Class: Thm (Kesten) $p_c(2) = \frac{1}{2}$ (We'll use Zhang's argument)